

UNIFIED FIELD EQUATIONS COUPLING FOUR FORCES AND PRINCIPLE OF INTERACTION DYNAMICS

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ABSTRACT. The main objective of this article is to postulate a principle of interaction dynamics (PID) and to derive unified field equations coupling all four forces. PID is a least action principle subject to div_A -free constraints for the variational element with A being gauge fields. The unified field equations of the coupled interactions of four forces are then uniquely derived based on 1) the Einstein principle of general relativity (or Lorentz invariance) and the principle of equivalence, 2) the principle of gauge invariance, and 3) the PID. The unified model gives rise to a complete new mechanism for spontaneously breaking gauge-symmetries and for energy and mass generation, which provides similar outcomes as the Higgs mechanism. For the electromagnetic and weak interactions alone, we derive a totally different electroweak theory, producing the three vector bosons W^\pm and Z , as well as three Higgs bosons—two charged and one neutral. One important outcome of the unified field equations is a natural duality between the interacting fields (g, A, W^a, B^b) , corresponding to graviton, photon, vector bosons W^\pm and Z and gluons, and the adjoint fields $(\Phi_\mu, \phi^0, \phi_W^a, \phi_B^b)$, which are bosonic fields. The interaction of the bosonic particle field Φ and graviton leads to a unified theory of dark matter and dark energy and explains the acceleration of expanding universe.

CONTENTS

1. Introduction	2
2. Orthogonal Decomposition and Variations with div_A -Free Constraint	4
2.1. Orthogonal Decomposition Theorems	4
2.2. Variations with div_A -free constraint	6
3. Principle of Interaction Dynamics	8
4. Unified Field Equations Coupling Four Forces	9
5. Gravitational Field Equations	12
6. Electroweak Theory	13
6.1. Weinberg-Salam Model	13
6.2. Electroweak theory based on Principle of Interaction Dynamics	16
References	17

Key words and phrases. four interacting forces, unified field equations, Principle of Interaction Dynamics, electroweak theory, quantum field theory, Higgs mechanism, dark energy, dark matter.

The work was supported in part by the Office of Naval Research, by the US National Science Foundation, and by the Chinese National Science Foundation.

1. INTRODUCTION

As we know there are four forces/interactions in nature: the electromagnetic force, the strong force, the weak force and the gravitational force. There are successful theories to describe these interactions, including the Einstein general theory of relativity for gravitation, the quantum electromagnetic dynamics (QED) for electromagnetism, the Weinberg-Salam electroweak theory unifying the weak and electromagnetic interactions [1, 7, 6], and the quantum chromodynamics (QCD) for the strong interactions. In particular, the Standard Model, a $SU(3) \otimes SU(2) \otimes U(1)$ gauge theory, provides an attempt for all known interactions except gravity; see among many others [4].

This article is an attempt to derive unified field equations coupling all four interactions. Apparently the unified field equations will be built upon the success of gauge theory and the Einstein general theory of relativity.

Motivated by the Higgs mechanism [3] of electroweak theory and the new gravitational field equations incorporating dark energy and dark matter [4, 5], the starting point of the study is to postulate a new least action principle for actions subject to energy-momentum conservation constraints, which we call the Principle of Interaction Dynamics (PID); see Section 3 for the precise statement of this principle.

The main motivation of postulating PID is as follows. Due to the presence of dark energy and dark matter, the energy-momentum tensor T_{ij} of normal matter is no longer conserved:

$$D^i(T_{ij}) \neq 0.$$

Therefore by an orthogonal decomposition theorem [5], the energy-momentum tensor T_{ij} can be decomposed naturally as

$$\begin{aligned} T_{ij} &= \tilde{T}_{ij} - \frac{c^4}{8\pi G} D_i D_j \varphi, \\ D^i \tilde{T}_{ij} &= 0, \end{aligned}$$

where φ is a scalar function defined on the space-time manifold. On the other hand, the Euler-Lagrangian of the scalar curvature part of the Einstein-Hilbert functional is divergence-free due to the Bianchi identity:

$$D^i(R_{ij} - \frac{1}{2}g_{ij}R) = 0.$$

Hence the modified gravitational field equations become

$$R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi G}{c^4} \tilde{T}_{ij},$$

which are equivalent to

$$(1.1) \quad R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi G}{c^4} T_{ij} - D_i D_j \varphi.$$

The derivation here is equivalent to studying the least action of the Einstein-Hilbert functional under divergence-free constraint for the variational element. To see this, consider the Einstein-Hilbert functional L_{EH} given by

$$L_{EH} = \int_M \left(R + \frac{8\pi G}{c^4} S \right) \sqrt{-g} dx.$$

Then PID amounts to saying that for any $X = \{X_{ij}\}$ with $D_g^i X_{ij} = 0$,

$$\lim_{\lambda \rightarrow 0} \frac{1}{\lambda} [L_{EH}(g_{ij} + \lambda X_{ij}) - L_{EH}(g_{ij})] = (\delta L_{EH}(g_{ij}), X) = 0.$$

By the orthogonal decomposition again,

$$\delta L_{EH}(g_{ij}) = -D_i D_j \varphi,$$

which is exactly the gravitational field equations (1.1), taking into consideration of the presence of dark matter and dark energy.

In order to take into account all four interactions, the natural constraint is then the div_A -free constraint, leading to the PID. Here A represent the gauge fields.

The unified field equations of the coupled interactions of four forces are then uniquely derived based on the following principles:

- the Einstein principle of general relativity (or Lorentz invariance) and the principle of equivalence, which amount to saying that the space-time is a 4-dimensional Riemannian manifold (M, g_{ij}) ,
- the principle of gauge invariance [8],
- the principle of interaction dynamics.

The unified field equations are given by (4.4)-(4.10). Here are a few main ingredients of these field equations:

FIRST, the Lagrangian action functional L is naturally the sum of three important parts as given in (4.2): the classical Einstein-Hilbert functional \mathcal{L}_{EH} , the Glashow-Weinberg-Salam functional \mathcal{L}_{GWS} without Higgs field terms, and the standard QCD functional \mathcal{L}_{QCD} . This Lagrangian action contains no tuning terms and is uniquely determined.

SECOND, thanks to the div_A -free constraint, the Euler-Lagrangian of the action functional is balanced by gradient fields, resulting the unified field equations coupling all four forces.

THIRD, the gradient fields break spontaneously the gauge-symmetries. This gives rise to a complete new mechanism for spontaneously breaking gauge-symmetries and for energy and mass generation, which provides similar outcomes as the Higgs mechanism.

FOURTH, one natural outcome of the new mechanism is the adjoint fields Φ_μ , ϕ^0 , ϕ_W^a , ϕ_B^b . Amazingly, there is a natural duality between the interacting fields (g, A, W^a, B^b) and the adjoint fields $(\Phi_\mu, \phi^0, \phi_W^a, \phi_B^b)$:

$$(1.2) \quad \begin{aligned} \{g_{\mu\nu}\} &\longleftrightarrow \Phi_\mu, \\ A_\mu &\longleftrightarrow \phi^0, \\ W_\mu^a &\longleftrightarrow \phi_W^a \quad \text{for } a = 1, 2, 3, \\ B_\mu^b &\longleftrightarrow \phi_B^b \quad \text{for } b = 1, \dots, 8. \end{aligned}$$

On the left hand side of this duality, the interacting fields represent the following particles:

$\{g_{\mu\nu}\}$	represents the graviton g ,
A_μ	represents the photon γ ,
W_μ^a	represents the vector bosons W^\pm and Z ,
B_μ^b	represents the gluons g_b .

On the right hand side of the duality (1.2), each field is a bosonic field: Φ represents a vector bosonic field, and ϕ^0 , ϕ_W^a , ϕ_B^b represent scalar bosonic fields.

FIFTH, the combination of the three Higgs bosonic fields on the right hand side of the duality induces three Higgs boson particles given by

$$\phi^\pm = \frac{1}{\sqrt{2}}(\phi_W^1 \pm i\phi_W^2), \quad \phi_W^3$$

with ϕ^\pm having \pm electric charges, and with ϕ_W^3 being neutral. Note that the classical Weinberg-Salam theory induces one neutral Higgs boson particle. All three Higgs bosons deduced here possess masses, generated by the new mechanism.

In fact, consider only the electromagnetic and weak interactions and ignore the effect of other interactions, we derive a totally different electroweak theory. Again this electroweak theory produces the three vector bosons W^\pm and Z , as well as the three Higgs bosons. The spontaneous gauge-symmetry breaking is achieved by the constraint action without Higgs terms in the action functional.

SIXTH, the new vector particle field Φ corresponding to the gravitational field $\{g_{ij}\}$ is massless with spin $s = 1$. This particle field corresponds to the scalar potential field, caused by the non-uniform distribution of matter in the universe as explained in [5]. In other words, the theory here demonstrates that the interaction between this particle field Φ and the graviton leads to a unified theory of dark matter and dark energy and explains the acceleration of expanding universe.

SEVENTH, the field ϕ^0 , adjoint to the electromagnetic potential A_μ , is a massless field with spin $s = 0$, and it needs to be confirmed experimentally. We think it is this particle field ϕ^0 that causes the vacuum fluctuation or zero-point energy.

EIGHTH, we conjecture that the combination of the eight scalar bosons ϕ_B^b corresponding to gluons g_b ($1 \leq b \leq 8$) leads to the Eightfold Way mesons: π^+ , π^- , π^0 , K^+ , K^- , K^0 , \bar{K}^0 , η^0 .

2. ORTHOGONAL DECOMPOSITION AND VARIATIONS WITH div_A -FREE CONSTRAINT

In this section, we address 1) the orthogonal decomposition of tensor fields with respect to gauge-connections $D + A$, and 2) the variations of functionals with div_A -free constraint. They are crucial for the derivation of the unified field equations of coupled interactions of all forces in the subsequent sections.

2.1. Orthogonal Decomposition Theorems. In [5], we obtained some orthogonal decomposition results for general tensor fields on a Riemannian manifold. For convenience, we briefly recall them in the following.

Theorem 2.1 (Ma and Wang [5]). *Let (M, g_{ij}) be a Riemannian manifold (including the Minkowski type manifolds) without boundary, and $u \in L^2(T_s^r M)$ with $r + s \geq 1$, where $L^2(T_s^r M)$ is the space consisting of all (r, s) -tensor fields on M which are square integrable. Then the tensor field u can be orthogonally decomposed into*

$$(2.1) \quad u = D\varphi + v, \quad \operatorname{div} v = 0,$$

for some $\varphi \in H^1(T_s^{r-1} M)$ or $\varphi \in H^1(T_{s-1}^r M)$, i.e.

$$(2.2) \quad \int_M D\varphi \cdot v \sqrt{g} dx = 0, \quad g = \det(g_{ij}).$$

Equality (2.2) is ensured by the formula

$$\int_M D\varphi \cdot v \sqrt{g} dx = - \int_M \varphi \cdot \operatorname{div} v \sqrt{g} dx.$$

Remark 2.1. *Theorem 2.1 implies that if a tensor field $u \in L^2(T_s^r M)$ satisfies*

$$(2.3) \quad \int_M u \cdot X \sqrt{g} dx = 0 \quad \forall X \in L^2(T_s^r M) \text{ with } \operatorname{div} X = 0,$$

then there exist a tensor field φ such that

$$(2.4) \quad u = D\varphi.$$

Based on the orthogonal properties (2.1)-(2.4), we can derive the follow scalar potential theorem.

Theorem 2.2 (Ma and Wang [5]). *Assume that the first Betti number of M is zero, i.e. any loop in M can shrink to a point. Let F be a functional of the Riemannian metric $\{g_{ij}\}$. If $\{g_{ij}\}$ is an extremum point of F with the divergence-free constraint, i.e.*

$$\int_M \delta F(g_{ij}) \cdot X \sqrt{g} dx = \frac{d}{d\lambda} F(g_{ij} + \lambda X_{ij})|_{\lambda=0} = 0$$

for all $X = \{X_{ij}\} \in L^2(T_2^0 M)$ and $\operatorname{div}_g X = 0$, where div_g is under the Levi-Civita connection of $\{g_{ij}\}$, then there is a scalar function $\varphi \in H^2(M)$ such that the metric $\{g_{ij}\}$ satisfies that

$$(2.5) \quad \delta F(g_{ij}) = D^2 \varphi.$$

We now generalize the above decomposition theorems to gauge connections, which are crucial to establish the dynamical theory of interaction fields. Hereafter we still assume that (M, g_{ij}) is an n -dimensional Riemannian manifold without boundary, where the metric $\{g_{ij}\}$ includes the Minkowski type Riemann metric, i.e., with a proper coordinate system, $\{g_{ij}\}$ can be expressed as in the form

$$(2.6) \quad \begin{pmatrix} -1 & 0 \\ 0 & G \end{pmatrix}$$

where G is a positive symmetric $(n-1) \times (n-1)$ matrix.

Let $u \in L^2(T_s^r M)$. We define the operators D_A and div_A as

$$(2.7) \quad D_A u = Du + u \otimes A,$$

$$(2.8) \quad \operatorname{div}_A u = \operatorname{div} u - A \cdot u,$$

where A is a vector or covector field, D and div are the usual gradient and divergent operators. It is clear that for D_A and div_A we have the integral formula

$$(2.9) \quad \int_M D_A u \cdot v \sqrt{g} dx = - \int_M u \cdot \text{div}_A v \sqrt{g} dx.$$

Then, for the differential operators given by (2.7) and (2.8) we have the following orthogonal decomposition theorem.

Theorem 2.3. *Let A is a given vector field or covector field, and $u \in L^2(T_s^r M)$ with $r + s \geq 1$. Then u can be orthogonally decomposed into*

$$(2.10) \quad u = D_A \varphi + v \quad \text{with} \quad \text{div}_A v = 0,$$

where $\varphi \in H^1(T_s^{r-1} M)$ or $\varphi \in H^1(T_{s-1}^r M)$.

The proof of Theorem 2.3 is the same as that of Theorem 2.1; see Ma and Wang [5], and we omit the details.

A few remarks are now in order.

First, Theorem 2.3 is a generalized version of Theorem 2.1, and is reduced to Theorem 2.1 if $A = 0$.

Second, when M is a compact manifold and (g_{ij}) is a positive matrix different from that given in (2.6), then u can be further orthogonally decomposed into

$$\begin{aligned} u &= D_A \varphi + v + h, \\ \text{div}_A v &= 0, \quad D_A h = 0, \quad \text{div}_A h = 0, \end{aligned}$$

where h is a harmonic field, and the space

$$\mathcal{H} = \{h \in L^2(T_s^r M) \mid D_A h = 0, \text{div}_A h = 0\}$$

is of finite dimensional.

Third, the orthogonal decomposition in Theorem 2.3 is very important for the interaction field theory developed in this article, because the vector fields A in (2.7) and (2.2) represent gauge fields in the interaction field equations, and lead to a new energy and mass generation mechanism, different from the famous Higgs mechanism.

2.2. Variations with div_A -free constraint. According to Theorem 2.3, we generalize the scalar potential theorem (Theorem 2.2) to the case where the variations of functionals F are under the div_A -free constraint.

Let $F = F(u)$ be a functional of tensor fields u . We say that an extremum point u_0 of $F(u)$ is under the div_A -free constraint, if

$$(2.11) \quad (\delta F(u_0), X) = \frac{d}{d\lambda} F(u_0 + \lambda X) = \int_M \delta F(u_0) \cdot X \sqrt{-g} dx = 0 \quad \forall \text{div}_A X = 0.$$

In particular if F is a functional of Riemannian metric g_{ij} , i.e. $u_0 = g_{ij}^0$ a Riemannian metric, then the differential operator D_A in $\text{div}_A X$ in (2.11) is

$$D_A^i = D^i + A^i \quad \text{or} \quad (D_A)_i = D_i + A_i.$$

and the connection in D is respect to the extremum point $u_0 = g_{ij}^0$.

Then we have the following theorems for the div_A -free constraint variations.

Theorem 2.4. *Let $F = F(g_{ij})$ be a functional of the Riemannian metric. Then there is a vector field $\Phi \in L^2(TM)$ such that the extremum points $\{g_{ij}\}$ of F with the div_A -free constraint for vector field A satisfy the equations*

$$(2.12) \quad \delta F(g_{ij}) = D\Phi + A \otimes \Phi.$$

In particular, if the vector field $A = 0$, then the conclusions of Theorem 2.2 hold true.

Theorem 2.5. *Let $F = F(u)$ be a functional of a vector field u . Then there is a scalar function $\varphi \in H^1(M)$ such that for given vector field A , the extremum points u of F with the div_A -free constraint satisfy the equation*

$$(2.13) \quad \delta F(u) = D\varphi + \varphi A.$$

Proof of Theorems 2.4 and 2.5. First, we verify Theorem 2.4. By (2.11), the extremum points $\{g_{ij}\}$ of F with div_A -free constraint satisfy

$$\int_M \delta F(g_{ij}) \cdot X \sqrt{-g} dx = 0 \quad \forall X \in L^2(T_0^2 M) \text{ with } \text{div}_A X = 0.$$

It implies that

$$(2.14) \quad \delta F(g_{ij}) \perp L_D^2(T_2^0 M) = \{v \in L^2(T_2^0 M) \mid \text{div}_A v = 0\}.$$

By Theorem 2.3, $L^2(T_2^0 M)$ can be orthogonally decomposed into two parts

$$\begin{aligned} L^2(T_2^0 M) &= L_D^2(T_2^0 M) \oplus G^2(T_2^0 M), \\ G^2(T_2^0 M) &= \{D_A B \mid B \in H^1(T_1^0 M)\}. \end{aligned}$$

Hence, it follows from (2.14) that

$$\delta F(g_{ij}) \in G^2(T_2^0 M),$$

which means the equality (2.12) holds true.

We are now in position to prove Theorem 2.5. Similarly, for the extremum vector fields u of F with div_A -free constraint satisfy

$$(2.15) \quad \int_M \delta F(u) \cdot X \sqrt{-g} dx = 0 \quad \forall X \in L^2(TM), \quad \text{div}_A X = 0.$$

In addition, Theorem 2.3 means that

$$\begin{aligned} L^2(TM) &= L_D^2(TM) \oplus G^2(TM), \\ L^2(TM) &= \{v \in L^2(TM) \mid \text{div}_A v = 0\}, \\ G^2(TM) &= \{D_A \varphi \mid \varphi \in H^1(M)\}. \end{aligned}$$

We infer from (2.15) that

$$\delta F(u) \in G^2(TM).$$

Theorem 2.5 follows, and the proof is complete. \square

3. PRINCIPLE OF INTERACTION DYNAMICS

We know that there are four fundamental forces in nature: the gravitational force, the electromagnetic force, the weak and the strong forces. There are successful theories to describe these interactions, including the Einstein general theory of relativity for gravitation, the QED for electromagnetism, the Weinberg-Salam electroweak theory, and QCD for the strong interactions. In particular, the Standard Model, a $SU(3) \otimes SU(2) \otimes U(1)$ gauge theory, can rather successfully describe all known interactions except gravity.

However, there is still no ideal unified theory to explain all four forces. For this purpose, we propose a basic principle, which we call Principle of Interaction Dynamics (PID). This principle is based on the variational theory developed in the previous section, and can be considered as a revised version of the Lagrangian least action principle for interacting fields.

Principle of Interaction Dynamics (PID). *For all physical interactions there are Lagrangian actions*

$$(3.1) \quad L(g, A, \psi) = \int_M \mathcal{L}(g_{ij}, A, \psi) \sqrt{g} dx,$$

where $g = \{g_{ij}\}$ is the Riemann metric representing the gravitational potential, A is a set of vector fields representing the gauge potentials, and ψ are the wave functions of particles. The actions (3.1) satisfy the invariance of general relativity (or Lorentz invariance) and the gauge invariance. Moreover, the states (g, A, ψ) are the extremum points of (3.1) with the div_A -free constraint.

According the above principle and Theorems 2.4 and 2.5, the field equations with respect to the action (3.1) are given in the form

$$(3.2) \quad \frac{\delta}{\delta g^{ij}} L(g, A, \psi) = (D_i + \sum_{k=1}^N \alpha_k A_i^k) \Phi_j,$$

$$(3.3) \quad \frac{\delta}{\delta A_i^k} L(g, A, \psi) = (D_i + \sum_{j=1}^N \beta_{kj} A_i^j) \varphi^k,$$

$$(3.4) \quad \frac{\delta}{\delta \psi} L(g, A, \psi) = 0,$$

where A^1, \dots, A^N are the gauge vector fields for the electromagnetic, weak, and strong interactions, $\Phi = (\Phi_1, \dots, \Phi_j)$ is a vector field induced by gravitational interaction, φ^k are scalar fields generated from the gauge field $A^k = (A_1^k, \dots, A_n^k)$, and α_k, β_{kj} ($1 \leq k, j \leq N$) are coupling parameters, which may depend on physical quantities as energy densities.

Now, we need to give some explanations for these field equations (3.2)-(3.4).

1. As explained in the Introduction, there are several reasons behind the div_A -free constraint. First, the coupling energies of interactions do not usually possess variational structure. In other words, the terms in the right-hand sides of (3.2) and (3.3) cannot be put into the Lagrangian. Second, the constraints eliminate the extra freedoms induced by the symmetry-invariance of Lorentz or general relativity and the gauge invariance. Hence each of ϕ^k

determines a gauge. Third, the div_A -constraint can be regarded as imposing conservations of energy-momentum.

2. The equations (3.2)-(3.4) preserve covariance of Lorentz or general relativity. However they break the gauge symmetry, caused by the constrained variational principle. In fact, it is utterly important that the field equations must break the gauge symmetry to avoid the existence of infinite number of solutions. However the action functional must preserve the gauge symmetry. Hence the least action must be taken with constraints, leading naturally to spontaneous gauge-symmetry breaking.
3. The spontaneous gauge-symmetry breaking caused by the terms $\alpha_k A_i^k \Phi_j$ and $\beta_{ij} A_i^k \varphi^k$ in (3.2) and (3.3) leads to energy creation. Also, for the weak interaction, the gauge-symmetry breaking gives rise to a new mass generation mechanism, different from the classical Higgs mechanism.
4. The fields g, A, ψ together with Φ and φ^k as unknowns of (3.2)-(3.4) characterize the interacting states. The fields Φ and φ^k are called the adjoint fields of g and A^k respectively.
5. The coupling parameters α_k represents the coupling strength between the gravitation and the gauge field A^k , and β_{kj} represents the coupling strength of A^k and A^j . They are determined by physical experiments and physical laws. These parameters are natural and required due to the energy-coupling of different fields.

4. UNIFIED FIELD EQUATIONS COUPLING FOUR FORCES

In this section we derive a unified model for all four interacting forces, based on the Principle of Interacting Dynamics. The action functional is the natural combination of the Einstein-Hilbert functional, the electroweak action, and the QCD action, and is given in the following form:

$$(4.1) \quad L = \int [\mathcal{L}_{EH} + \mathcal{L}_{GWS} + \mathcal{L}_{QCD}] \sqrt{-g} dx,$$

where

$$(4.2) \quad \begin{aligned} \mathcal{L}_{EH} &= R + \frac{8\pi G}{c^4} S, \\ \mathcal{L}_{GWS} &= -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &\quad + \sum_{k=1}^6 \left\{ \bar{L}_k (i\gamma^\mu \tilde{D}_\mu - m_k) L_k + \bar{\psi}_k^R (i\gamma^\mu \tilde{D}_\mu - m_k^l) \psi_k^R \right\}, \\ \mathcal{L}_{QCD} &= -\frac{1}{4} F_{\mu\nu}^b F^{b\mu\nu} + \sum_{k=1}^6 \bar{q}_k (i\gamma^\mu \tilde{D}_\mu - m_k^q) q_k, \end{aligned}$$

where R is the scalar curvature of spacetime Riemannian manifold, S is the momentum density, W_μ^a ($a = 1, 2, 3$) are the electroweak gauge potentials, A_μ is the

electromagnetic potential, B_μ^b ($1 \leq b \leq 8$) are the color gauge potentials, and

$$\begin{aligned}
(4.3) \quad & W_{\mu\nu}^a = D_\mu W_\nu^a - D_\nu W_\mu^a + g_1 f^{abc} W_\mu^b W_\nu^c, \\
& F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu, \\
& F_{\mu\nu}^b = D_\mu B_\nu^b - D_\nu B_\mu^b + g_2 g^{bcd} B_\mu^c B_\nu^d, \\
& \tilde{D}_\mu L_k = (\nabla_\mu - i \frac{g_1}{2} W_\mu^a \sigma_a + i \frac{e}{2} A_\mu) L_k, \\
& \tilde{D}_\mu \psi_k^R = (\nabla_\mu + i e A_\mu) \psi_k^R, \\
& \tilde{D}_\mu q_k = (\nabla_\mu + i \frac{g_2}{2} B_\mu^b \lambda_b) q_k,
\end{aligned}$$

where g_1 and g_2 are constants, e is the charge of an electron, f^{abc} and g^{bcd} are the structure constants of $SU(2)$ and $SU(3)$ respectively, σ_a ($a = 1, 2, 3$) are the Pauli matrices, λ_b ($1 \leq b \leq 8$) are the Gell-Mann matrices, $L_k = (\psi_{\nu k}, \psi_k^L)^t$ are the wave functions of left-hand lepton and quark pairs (each has 3 generations), ψ_k^R are the right-hand leptons and quarks, $q_k = (q_{k1}, q_{k2}, q_{k3})^t$ is the k -th flavored quark, D_μ is the general covariant derivative, and ∇_μ is the Lorentz Vierbein covariant derivative [4].

For the gauge fields V_μ , we have

$$D_\mu V_\nu - D_\nu V_\mu = \partial_\mu V_\nu - \partial_\nu V_\mu.$$

By (3.2) and (3.3), it follows from (4.3) that the field equations coupling all four interactions are given as follows:

$$\begin{aligned}
(4.4) \quad & R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{8\pi G}{c^4} T_{\mu\nu} \\
& = (D_\mu + \alpha_0 A_\mu + \alpha_{W_a} W_\mu^a + \alpha_{B_j} B_\mu^j) \Phi_\nu,
\end{aligned}$$

$$\begin{aligned}
(4.5) \quad & \partial^\mu (\partial_\mu A_\nu - \partial_\nu A_\mu) - e \sum_{k=1}^6 \left[\frac{1}{2} \bar{L}_k \gamma_\nu L_k + \bar{\psi}_k^R \gamma_\nu \psi_k^R \right] \\
& = (D_\nu + \beta_0 A_\nu + \beta_{W_a} W_\nu^a + \beta_{B_j} B_\nu^j) \phi^0,
\end{aligned}$$

$$\begin{aligned}
(4.6) \quad & \partial^\mu W_{\mu\nu}^a - \frac{g_1}{2} \sum_{b=1}^3 f^{bca} g^{\mu\alpha} W_{\mu\nu}^a W_\alpha^c + \frac{g_1}{2} \sum_{k=1}^6 \bar{L}_k \gamma_\nu \sigma_a L_k \\
& = (D_\nu + \kappa_0^a A_\nu + \kappa_{W_b}^a W_\nu^b + \kappa_{B_j}^a B_\nu^j) \phi_W^a,
\end{aligned}$$

$$\begin{aligned}
(4.7) \quad & \partial^\mu F_{\mu\nu}^b - \frac{g_2}{2} \sum_{k=1}^8 g^{klb} g^{\mu\alpha} B_{\mu\nu}^b B_\alpha^l - \frac{g_2}{2} \sum_{k=1}^6 \bar{q}_k \gamma_\nu \lambda_b q_k \\
& = (D_\nu + \theta_0^b A_\nu + \theta_{W_l}^b W_\nu^l + \theta_{B_l}^b B_\nu^l) \phi_B^b,
\end{aligned}$$

$$(4.8) \quad (i\gamma^\mu \tilde{D}_\mu - m_k) L_k = 0 \quad \text{for } k = 1, 2, 3,$$

$$(4.9) \quad (i\gamma^\mu \tilde{D}_\mu - m_k^l) \psi_k^R = 0 \quad \text{for } k = 1, 2, 3,$$

$$(4.10) \quad (i\gamma^\mu \tilde{D}_\mu - m_k^q) q_k = 0 \quad \text{for } k = 1, \dots, 8,$$

where

$$T_{\mu\nu} = \frac{\delta S}{\delta g_{ij}} + \frac{c^4}{16\pi G} g^{\alpha\beta} [W_{\alpha\mu}^a W_{\beta\nu}^a + F_{\alpha\mu} F_{\beta\nu} + F_{\alpha\mu}^b F_{\beta\nu}^b] - \frac{c^4}{16\pi G} g_{\mu\nu} (\mathcal{L}_{GWS} + \mathcal{L}_{QCD}).$$

Of course, the above field equations need to be supplemented with coupled gauge equations to fix the gauge. We think the gauge equations should be given by

$$(4.11) \quad D^\mu A_\mu = \text{constant}, \quad D^\mu W_\mu^a = \text{constant}, \quad D^\mu B_\mu^b = \text{constant}.$$

From the field equations (4.4)-(4.7), we see that there is a corresponding relation between the interacting fields (g, A, W^a, B^b) and the adjoint fields $(\Phi_\mu, \phi^0, \phi_W^a, \phi_B^b)$ given as follows:

$$(4.12) \quad \begin{aligned} \{g_{\mu\nu}\} &\longleftrightarrow \Phi_\mu, \\ A_\mu &\longleftrightarrow \phi^0, \\ W_\mu^a &\longleftrightarrow \phi_W^a \quad \text{for } a = 1, 2, 3, \\ B_\mu^b &\longleftrightarrow \phi_B^b \quad \text{for } b = 1, \dots, 8. \end{aligned}$$

In addition, we know that

$$\begin{aligned} \{g_{\mu\nu}\} &\text{ represents the graviton } g, \\ A_\mu &\text{ represents the photon } \gamma, \\ W_\mu^a &\text{ represents the vector bosons } W^\pm \text{ and } Z, \\ B_\mu^b &\text{ represents the gluons } g_b. \end{aligned}$$

Taking divergence on both sides of (4.4)-(4.6) respectively, we obtain that

$$\begin{aligned} D^\mu D_\mu \Phi + (\text{div} V_1) \Phi &= o(1), \\ D^\mu D_\mu \phi^0 + (\text{div} V_2) \phi^0 &= o(1), \\ D^\mu D_\mu \phi_W^a + (\text{div} V_3^a) \phi_W^a &= o(1) \quad \text{for } a = 1, 2, 3, \\ D^\mu D_\mu \phi_B^b + (\text{div} V_3^b) \phi_B^b &= o(1) \quad \text{for } b = 1, \dots, 8, \end{aligned}$$

which imply that Φ represents a vector boson, and $\phi^0, \phi_W^a, \phi_B^b$ represent scalar bosons. Here

$$\begin{aligned} V_1 &= \alpha_0 A_\mu + \alpha_{W^a} W_\mu^a + \alpha_{B^j} B_\mu^j, \\ V_2 &= \beta_0 A_\nu + \beta_{W^a} W_\nu^a + \beta_{B^j} B_\nu^j, \\ V_3^a &= \kappa_0^a A_\nu + \kappa_{W^b}^a W_\nu^b + \kappa_{B^j}^a B_\nu^j, \\ V_3^b &= \theta_0^j A_\nu + \theta_{W^b}^j W_\nu^b + \theta_{B^l}^j B_\nu^l. \end{aligned}$$

In other words, each of the fields $\{g_{\mu\nu}\}, A_\mu, W_\mu^a$ ($a = 1, 2, 3$) and B_μ^b ($b = 1, \dots, 8$) corresponds to a bosonic field. Therefore the corresponding relation (4.12) leads to the following conjectures:

- (1) There are three Higgs boson particles given by

$$\phi^\pm = \frac{1}{\sqrt{2}}(\phi_W^1 \pm i\phi_W^2), \quad \phi_W^3$$

with ϕ^\pm having \pm electric charges, and with ϕ_W^3 being neutral. Note that the classical Weinberg-Salam theory induces one neutral Higgs boson particle. All three Higgs bosons deduced here possess masses, generated by the new mechanism here, different from the classical Higgs mechanism.

- (2) The combination of the eight scalar bosons ϕ_B^b corresponding to gluons g_b ($1 \leq b \leq 8$) leads to the Eightfold Way mesons: $\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \bar{K}^0, \eta^0$.

- (3) The new particle field Φ corresponding to the gravitational field $\{g_{ij}\}$ is massless with spin $s = 1$. This particle field corresponds to the scalar potential field, caused by the non-uniform distribution of matter in the universe as explained in [5]. The interaction between this particle field Φ and the graviton leads to a unified theory of dark matter and dark energy and explains the acceleration of expanding universe. See also Conclusion 5.1 below.
- (4) The field ϕ^0 , adjoint to the electromagnetic potential A_μ , is a massless field with spin $s = 0$. Of course, the particle ϕ^0 , adjoint to photon γ , still needs to be confirmed experimentally. We think it is this particle field ϕ^0 that causes the vacuum fluctuation or zero-point energy.
- (5) From the mathematical point of view, the classical Einstein field equations contain 10 gravitational field equations solving for 6 unknown functions in g_{ij} due to coordinate transformations, leading to an ill-posed problem. This ill-posedness is resolved in the field equations (4.4)-(4.10) due to the presence of the vector potential Φ_μ derived from the variation with constraint.

5. GRAVITATIONAL FIELD EQUATIONS

We know that the action of gravitation is the well known Einstein-Hilbert functional given by

$$(5.1) \quad L = \int_M \left[R + \frac{8\pi G}{c^4} S \right] \sqrt{-g} dx.$$

When the other three interacting forces are very weak, the gravitational field equations decouple with the gauge fields $A = \{A^1, \dots, A^N\}$. In this case the adjoint field Φ of the metric g in (3.2) becomes a gradient field of a scalar function as demonstrated by the authors in [5]:

$$\Phi = -D\varphi.$$

Namely the decoupled gravitational field equations for (5.1) are taken in the following form

$$(5.2) \quad R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi G}{c^4}T_{ij} - D_i D_j \varphi.$$

The interested readers are referred to [5] for more detailed discussions on the above modified gravitational field equations and their physical implications. In particular, the new field equations give rise to a unified theory for dark energy and dark matter, and provide an explanation for the Rubin rotational curves and for the acceleration of expanding galaxies.

In the case where the gauge fields A can not be ignored, the field equations (3.2) are expressed as

$$(5.3) \quad R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi G}{c^4}T_{ij} + D_i \Phi_j + \sum_{k=1}^N \lambda_k A_i^k \Phi_j$$

Since the strong and weak interactions are short range forces, in the large scale domains, only the electromagnetic field $A_\mu = (A_0, A_1, A_2, A_3)$ is retained in (5.3). Hence the large scale coupling interacting gravitational field equations are of the

following form

$$(5.4) \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu} + D_\mu\Phi_\nu + \lambda A_\mu\Phi_\nu,$$

where λ is a coupling parameter, and

$$(5.5) \quad \Phi_\mu = (\Phi_0, \Phi_1, \Phi_2, \Phi_3)$$

is the adjoint vector potential of gravitation, which is regarded as the effects induced by the coupling of matter and electromagnetic fields.

A direct consequence of (5.4)-(5.5) is as follows:

Conclusion 5.1. *There exists a field different from photon field, which is a vector boson Φ_μ as in (5.5), possessing zero mass and spin $s = 1$.*

To see this, let the momentum tensor $T_{\mu\nu} = 0$ (in vacuum), and taking divergence on both sides of (5.4), by $\text{div}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) = 0$, we have

$$(5.6) \quad D^\mu D_\mu\Phi_\nu + \lambda(D^\mu A_\mu\Phi_\nu + A_\mu \cdot D^\mu\Phi_\nu) = 0.$$

Since the fields A_μ and Φ_ν are weak and the electromagnetic gauge $\partial^\mu A_\mu = 0$, we can ignore the second order terms and infer from equations (5.6) that

$$(5.7) \quad D^\mu D_\mu\Phi = 0.$$

In weak gravitation, the operator $D^\mu D_\mu$ is approximately the Klein-Gorden operator. Hence (5.7) implies Conclusion 5.1.

As concluded in the previous section, the massless with spin $s = 1$ particle field Φ corresponds to the scalar potential field, caused by the non-uniform distribution of matter in the universe as explained in [5]. The interaction between this particle field Φ and the graviton explains the dark energy and dark matter.

6. ELECTROWEAK THEORY

6.1. Weinberg-Salam Model. We first recall the classical Weinberg-Salam electroweak theory, which is a $SU(2) \times U(1)$ gauge theory. The action density is given by

$$(6.1) \quad \mathcal{L} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_H,$$

where \mathcal{L}_G is the gauge part, \mathcal{L}_F is the fermionic part, and \mathcal{L}_H is the scalar Higgs sector. They are given as follows [4, 2]:

$$(6.2) \quad \begin{aligned} \mathcal{L}_G &= -\frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \\ \mathcal{L}_F &= i\bar{R}\gamma^\mu D_\mu R + i\bar{L}\gamma^\mu D_\mu L, \\ \mathcal{L}_H &= D_\mu\phi^\dagger D^\mu\phi - \lambda(\phi^\dagger\phi - a)^2 + G_e(\bar{L}\phi R + \bar{R}\phi^\dagger L), \end{aligned}$$

where $\gamma^\mu = (\gamma^0, \gamma^1, \gamma^2, \gamma^3)$ are the Dirac matrices, and

$$\begin{aligned} W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_1 f^{kij} W_\mu^i W_\nu^j, \\ F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ D_\mu R &= (\partial_\mu + ig_2 B_\mu)R, \\ D_\mu L &= (\partial_\mu + i\frac{g_2}{2}B_\mu - i\frac{g_1}{2}W_\mu^k \sigma_k)L, \\ D_\mu \phi &= (\partial_\mu - i\frac{g_2}{2}B_\mu - i\frac{g_1}{2}W_\mu^k \sigma_k)\phi, \end{aligned}$$

where g_1 and g_2 are the coupling constants, f^{kij} ($1 \leq k, i, j \leq 3$) are the structure constants of $SU(2)$, σ_k ($1 \leq k \leq 3$) are the Pauli matrices, and

$$\begin{aligned} W_\mu^k &= (W_0^k, W_1^k, W_2^k, W_3^k) \quad \text{for } k = 1, 2, 3, \\ B_\mu &= (B_0, B_1, B_2, B_3), \\ L &= \begin{pmatrix} \psi_{\nu_e}^L \\ \psi_e^L \end{pmatrix}, \quad R = \psi_e^R, \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \end{aligned}$$

where W_μ^k is the Yang-Mills gauge field corresponding to the k -th generator of $SU(2)$, B_μ is the gauge field with respect to $U(1)$, ψ_e^L and $\psi_{\nu_e}^L$ are the wave functions of left-hand electron and neutrino respectively, ψ_e^R is the wave function of right-hand electron, and ϕ is the Higgs scalar field.

The complexity of the Weinberg-Salam model (6.1) and (6.2) is due to the coupling to the Higgs field ϕ , which breaks the gauge symmetry, and creates mass. The basic idea, called the Higgs mechanism, is as follows. The action (6.1) is invariant under the $SU(2)$ gauge transformation:

$$\begin{aligned} L &\rightarrow e^{(i/2)\theta_k \cdot \sigma_k} L, \\ \phi &\rightarrow e^{-(i/2)\theta_k \cdot \sigma_k} \phi, \\ R &\rightarrow R, \\ W_\mu^k &\rightarrow W_\mu^k - \frac{2}{g_1} \partial_\mu \theta_k + f^{kij} \theta_i W_\mu^j, \\ B_\mu &\rightarrow B_\mu, \end{aligned} \tag{6.3}$$

and the $U(1)$ gauge transformation:

$$\begin{aligned} L &\rightarrow e^{(i/2)\beta} L \quad \beta \text{ a real function}, \\ \phi &\rightarrow e^{-(i/2)\beta} \phi, \\ R &\rightarrow e^{i\beta} R, \\ W_\mu^k &\rightarrow W_\mu^k - \frac{2}{g_2} \partial_\mu \beta, \\ B_\mu &\rightarrow B_\mu + \frac{2}{g_2} \partial_\mu \beta. \end{aligned} \tag{6.4}$$

Hence the variational equations

$$\frac{\delta}{\delta W} \tilde{L}(W, B, L, R, \phi) = 0 \tag{6.5}$$

are covariant for the action

$$\tilde{L} = \int [\mathcal{L}_G + \mathcal{L}_F] dx. \tag{6.6}$$

However, (6.5) are of the following form

$$\partial^\mu W_{\mu\nu}^k = o(W) \quad \text{the higher order terms of } W, \tag{6.7}$$

which implies that the field particles described by W^k are massless. This is a contradiction to the physical fact that the W^k particles have mass.

If one adds a term

$$m W_{\mu\nu}^k \cdot W^{k\mu\nu}$$

into the action \mathcal{L}_G , then the action

$$\mathcal{L}_G + mW_{\mu\nu}^k \cdot W^{k\mu\nu}$$

breaks the symmetry for the gauge transformation (6.3) and (6.4).

Higgs mechanism provides a solution when we add the Higgs action \mathcal{L}_H into (6.6). We infer then from (6.1) and (6.2) that

$$\phi_0 = \begin{pmatrix} 0 \\ a \end{pmatrix}$$

is an extremum point of (6.1). Hence

$$\Phi_0 = (0, 0, 0, 0, \phi_0)$$

is a solution of

$$\delta L = 0, \quad L = \int [\mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_H] dx.$$

Consider the translation for $\Phi = (W, B, L, R, \phi)$:

$$\Phi = \tilde{\Phi} + \Phi_0, \quad \tilde{\Phi} = (\tilde{W}, \tilde{B}, \tilde{L}, \tilde{R}, \tilde{\phi}).$$

Then the variational equations of L for $\tilde{\Phi}$ are given by

$$(6.8) \quad \begin{pmatrix} \frac{\delta}{\delta \tilde{W}} L \\ \frac{\delta}{\delta \tilde{B}} L \end{pmatrix} = \begin{pmatrix} \partial^\mu \tilde{W}_{\mu\nu} \\ \partial^\mu \tilde{F}_{\mu\nu} \end{pmatrix} + M \begin{pmatrix} \tilde{W}_\nu \\ \tilde{B}_\nu \end{pmatrix} + o(W, B) = 0,$$

where M is the mass matrix. The equations (6.8) show that the particles $(\tilde{W}_\nu, \tilde{B}_\nu)$ have masses due to the spontaneous gauge-symmetry breaking for the gauge transformations (6.3) and (6.4).

In the Weinberg-Salam model, there are three mass bosons Z_μ , W_μ^+ , W_μ^- and photon A_μ induced from W_μ^k and B_μ as follows:

$$\begin{aligned} Z_\mu &= \frac{1}{\sqrt{g_1^2 + g_2^2}} [g_1 W_\mu^3 + g_2 B_\mu] = \cos \theta_W W_\mu^3 + \sin \theta_W B_\mu, \\ A_\mu &= \frac{1}{\sqrt{g_1^2 + g_2^2}} [-g_2 W_\mu^3 + g_1 B_\mu] = -\sin \theta_W W_\mu^3 + \cos \theta_W B_\mu, \\ W_\mu^\pm &= \frac{1}{\sqrt{2}} [W_\mu^1 \pm i W_\mu^2], \end{aligned}$$

where the Weinberg angle θ_W is defined by

$$\cos \theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad \sin \theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}.$$

The values of the Weinberg angle θ_W and masses M_Z , M_W are determined as

$$(6.9) \quad \begin{aligned} \sin^2 \theta_W &= 0.2325 \pm 0.008, \\ M_Z &= 91.173 \pm 0.02 \text{ GeV}/c^2, \\ M_W &= 80.22 \pm 0.26 \text{ GeV}/c^2. \end{aligned}$$

6.2. Electroweak theory based on Principle of Interaction Dynamics.

There are two shortcomings for the classical Weinberg-Salam model. The first is its complexity, and the second is that the electromagnetism can not be decoupled in a natural and apparent manner.

In this section, we present a much simpler electroweak theory.

We take the action

$$(6.10) \quad L = \int \left[-\frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{L}(i\gamma^\mu D'_\mu - m)L + \bar{R}(i\gamma^\mu D_\mu - m_e)R \right] dx,$$

where

$$\begin{aligned} W_{\mu\nu}^k &= \partial_\mu W_\nu^k - \partial_\nu W_\mu^k + \lambda f^{kij} W_\mu^i W_\nu^j, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\ D'_\mu &= \partial_\mu + ieA_\mu + \lambda W_\mu^k \sigma_k, \\ D_\mu &= \partial_\mu + ieA_\mu. \end{aligned}$$

Obviously, for the case where there is no weak interaction present, $W_{\mu\nu}^k = 0$, $\psi_\nu^L = 0$, and the action (6.10) reduces to the classical QED action:

$$L_{QED} = \int \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_e(i\gamma^\mu D_\mu - m_e)\psi_e \right] dx.$$

By (3.3) and (3.4), from (6.10) we obtain the electroweak interacting field equations as follows:

$$(6.11) \quad \begin{aligned} \partial^\mu W_{\mu\nu}^k - \frac{\lambda}{2} \sum_{l=1}^3 f^{lik} W_{\mu\nu}^l W_\mu^i + J_\nu^k, \\ = (\partial_\nu + \alpha_k W_\nu^k + \alpha_0 A_\nu) \phi_1^k \quad \text{for } k = 1, 2, 3, \end{aligned}$$

$$(6.12) \quad \partial^\mu F_{\mu\nu} + J_\nu^L + J_\nu^R = (\partial_\nu + \beta_j W_\nu^j + \beta_0 A_\nu) \phi_2,$$

$$(6.13) \quad (i\gamma^\mu D_\mu - m)L = 0,$$

$$(6.14) \quad (i\gamma^\mu D_\mu - M_e)R = 0,$$

where

$$\begin{aligned} J_\nu^k &= i\lambda \bar{L} \gamma^\nu \sigma_k L, \\ J_L &= ie \bar{L} \gamma^\nu L, \\ J_R &= ie \bar{R} \gamma^\nu R. \end{aligned}$$

Let the scalar function ϕ_1^k be in the form

$$\phi_1 = \rho^k + \phi^k, \quad \rho^k = \text{constant}, \phi^k \simeq 0.$$

Then omitting higher order terms, equations (6.11) become

$$(6.15) \quad \partial^\mu W_{\mu\nu}^k - m_k W_\nu^k = 0, \quad m_k = \rho^k \alpha_k, \quad \text{for } k = 1, 2, 3.$$

Let

$$m_1 = m_2 = M_W, \quad m_3 = M_Z,$$

where M_W and M_Z are given by (6.9). Then we derive three vector bosons with respective masses as follows:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2),$$

$$Z_\mu = W_\mu^3,$$

which are the same as those derived from the Weinberg-Salam theory.

Taking divergence on both sides of (6.11) and noticing that $\partial^\nu \partial^\mu W_{\nu\mu}^k = 0$, we obtain that

$$(6.16) \quad \partial^\mu \partial_\mu \phi_1^k = m_k \phi_1^k + \text{higher order terms},$$

where m_k is the constant term in

$$\alpha_k \partial^\mu W_\mu^k + \alpha_0 \partial^\mu A_\mu = m_k + \text{higher order terms}.$$

Hence (6.16) can be regarded as the Higgs field equations, and ϕ_1^k are the Higgs particles with mass m_k .

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